Equivalent Frame Method

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Equivalent Frame Method

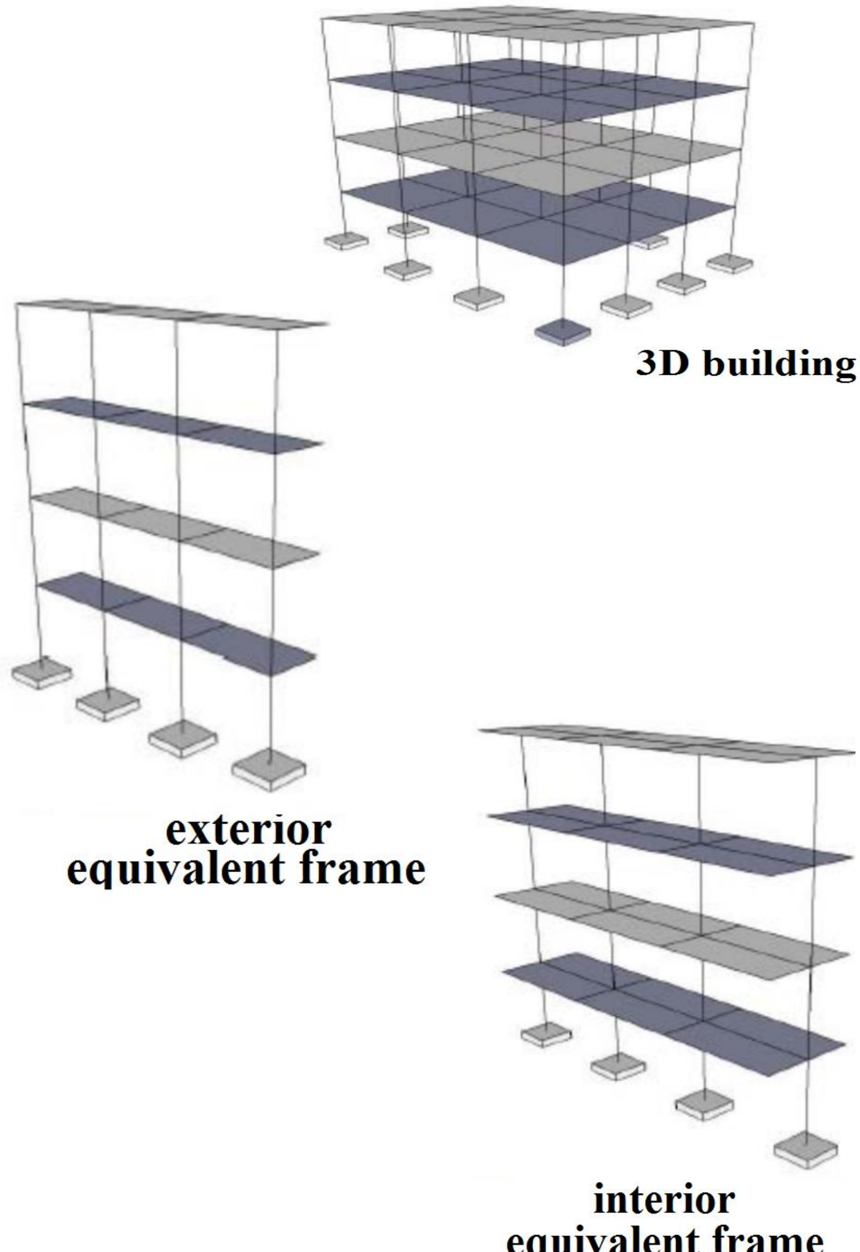
Dividing the building into equivalent frames, then analysing them either by (moment distribution method) or (slope deflection method). After that, distributing positive and negative moments to the middle and column strips will be in the same way that of D.D.M.

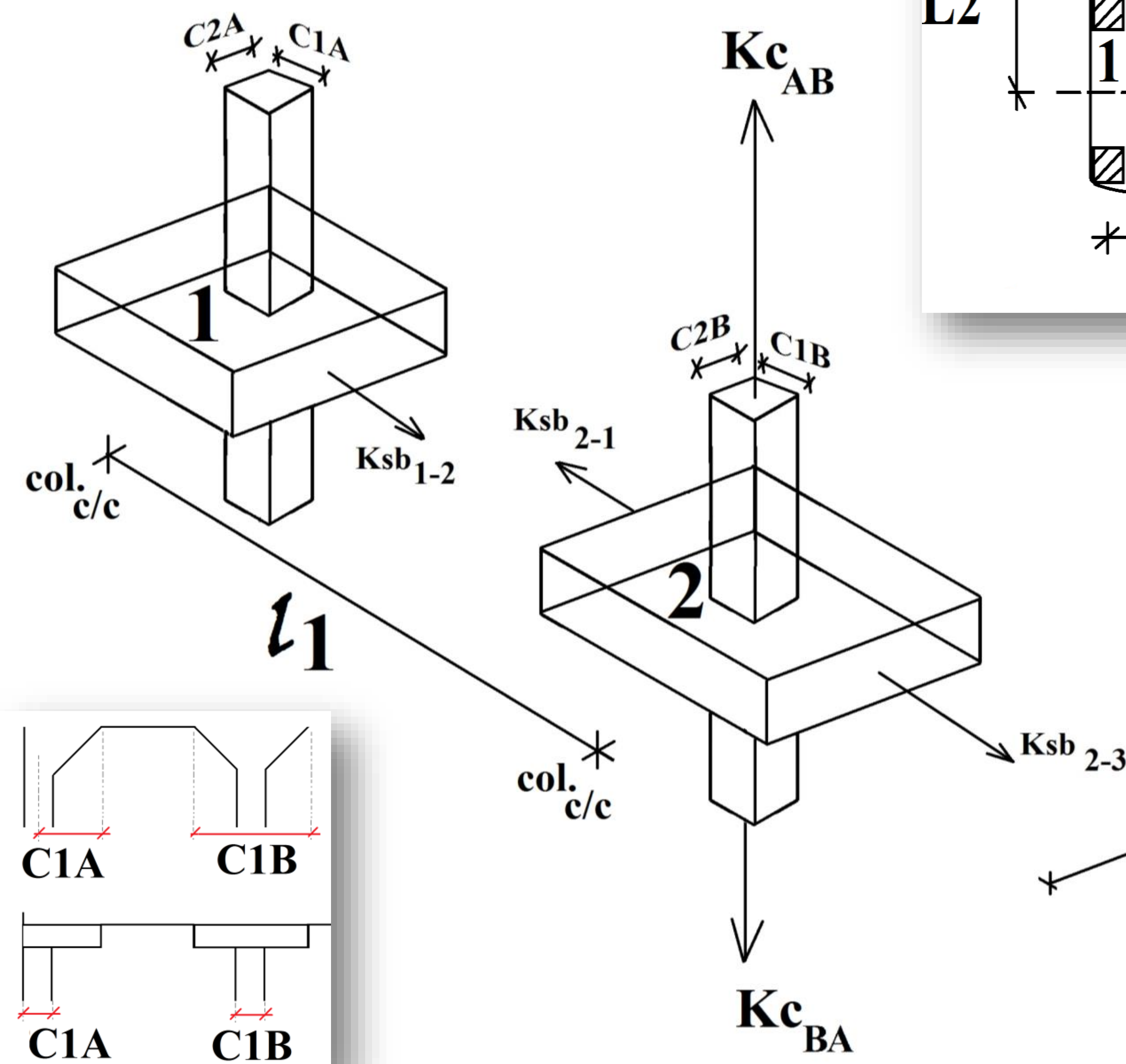
The followings should be identified:

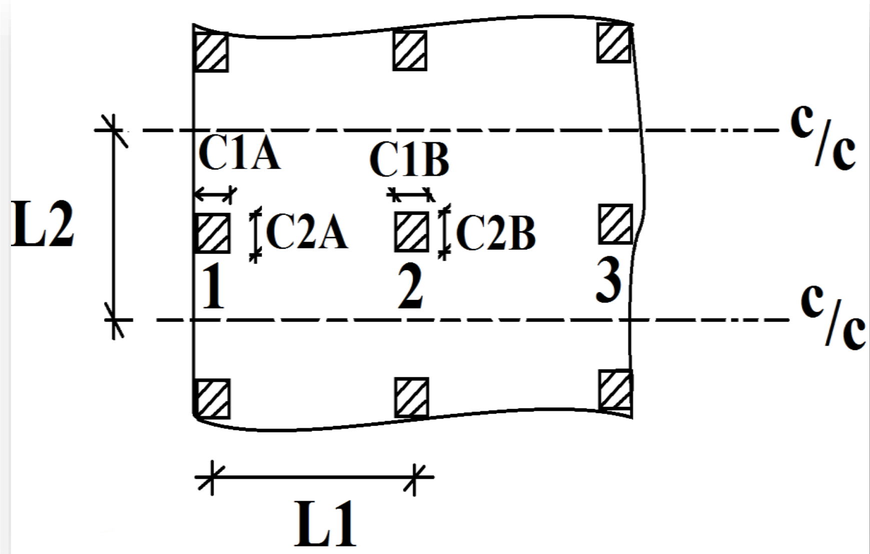
- *K, flexural stiffness* ($K = k\frac{E\,I}{L}$)*;*
- *COF, carryover factors;*
- *DF, distribution factors; and*
- *FEM, fixed-end moments.*

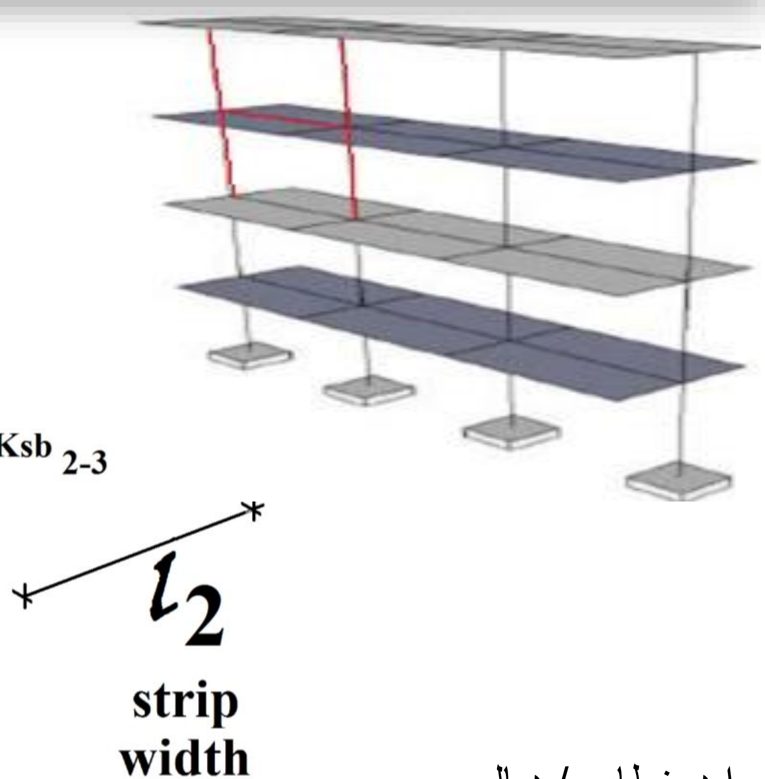
$$\mathrm{FEM} = \alpha\, \mathrm{W_u}\, \mathrm{l_2}\, \mathrm{l_1^2}$$

α=4 for prismatic members, otherwise tables should be used.









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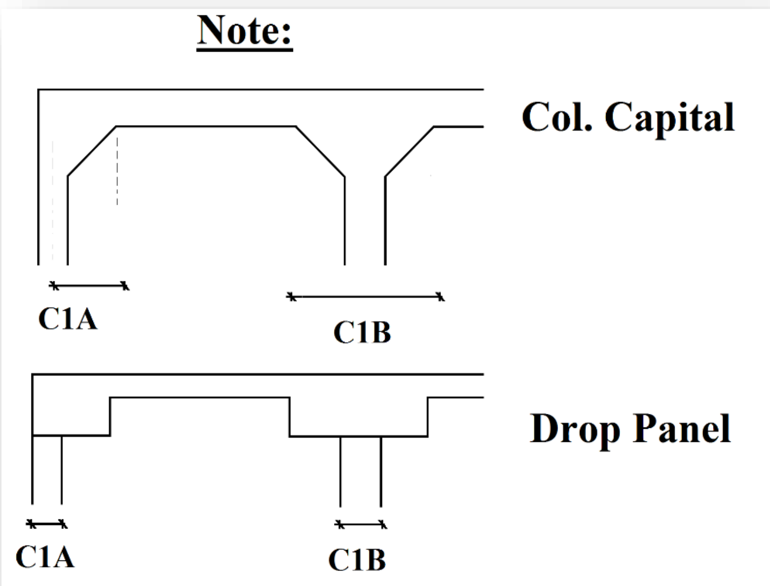
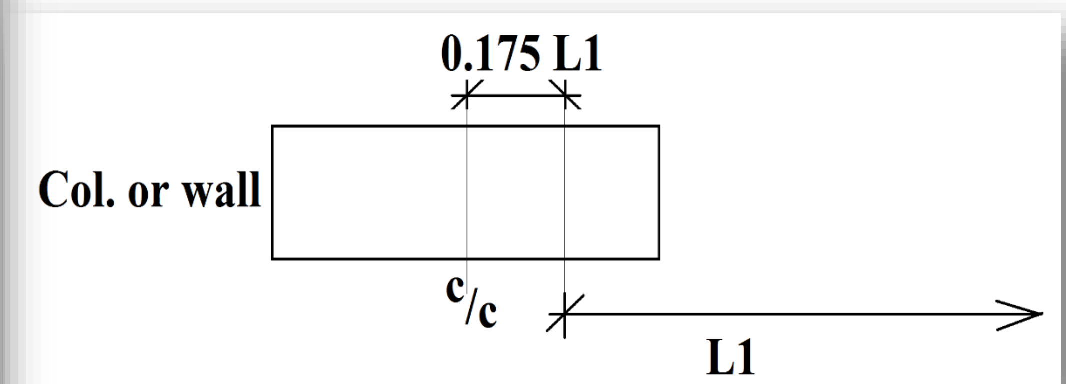
•Equivalent column has a less stiffness than that of the actual column, capital, bracket or wall. A beam is not considered a supporting member for the equivalent frame.

•Non-rectangular supports should be treated as equivalent square supports (having the same cross-sectional area).

•Negative factored moments for design must be taken at faces of rectilinear supports, but not at a distance greater than (0.175 L1) from the centre of a support. This absolute value is limited on long narrow supports in order to prevent undue reduction in design moment.





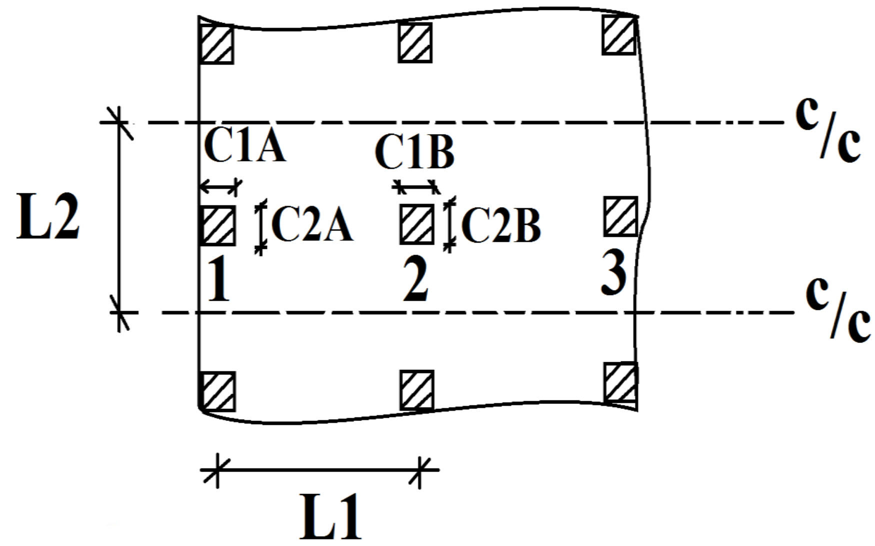
Flexural stiffness of slab-beam Ksb:

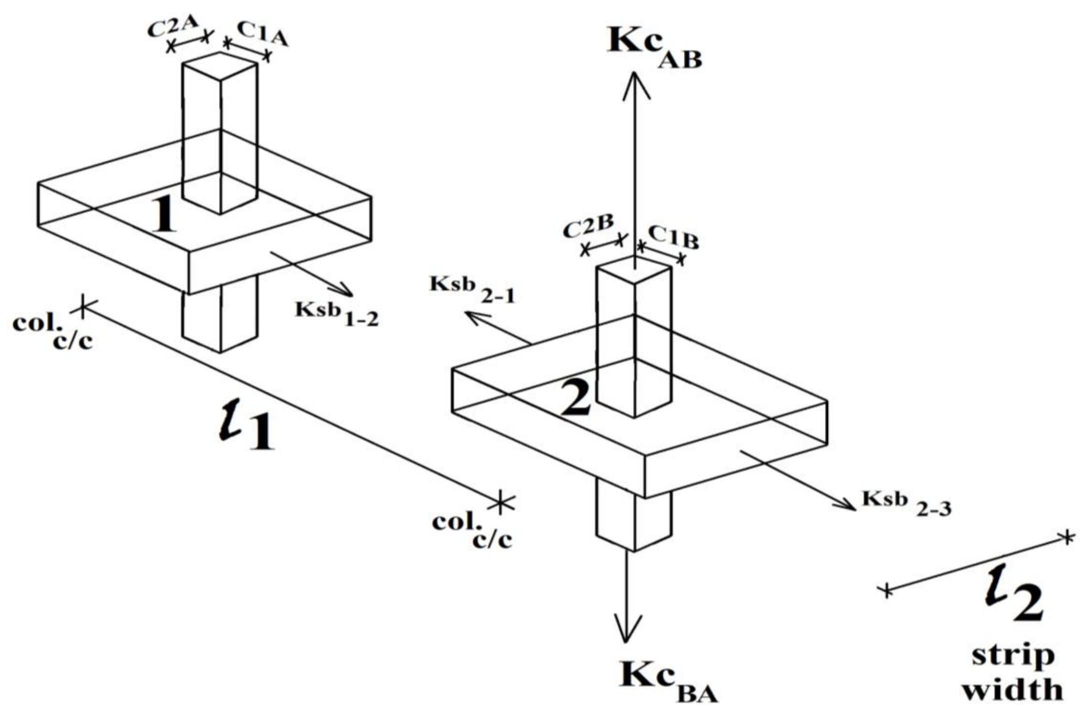
1. With<u>out</u> internal parallel beam:

$$Ksb_{1-2} = K_{AB}\frac{4700\sqrt{f'c}\ \ I_{cs}}{L1}$$

$$Ksb_{2-1} = K_{BA}\frac{4700\sqrt{f'c}\ \ I_{cs}}{L1}$$

$$I_{cs} = \frac{L2\ .\ h_f^3}{12}$$





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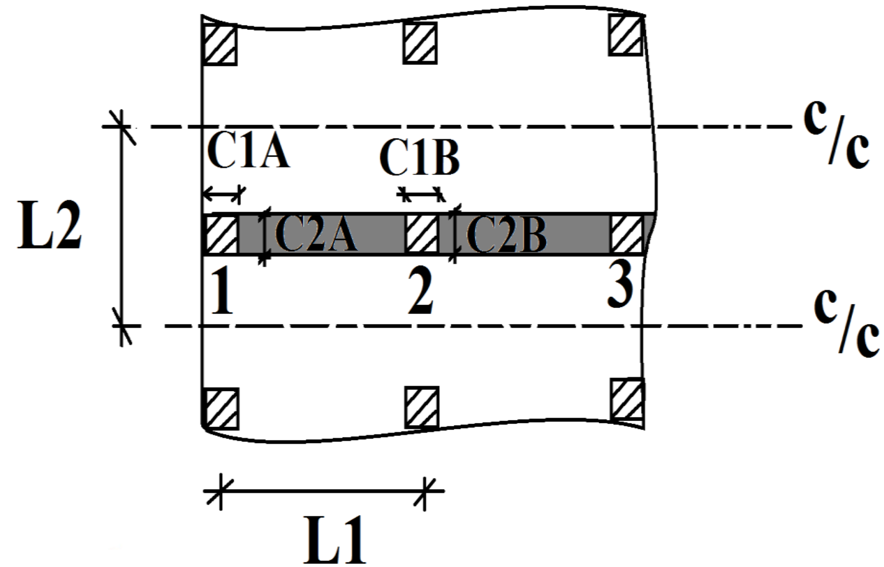
2. With internal parallel beam:

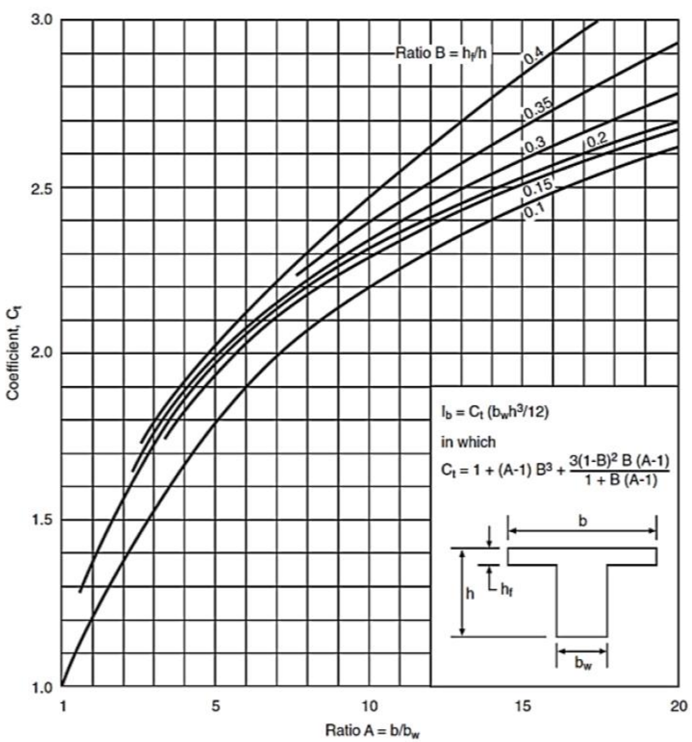
$$Ksb_{1-2} = K_{AB}\frac{4700\sqrt{f'c}\ \ (C_t\,\frac{b_w h^3}{12})}{L1}$$

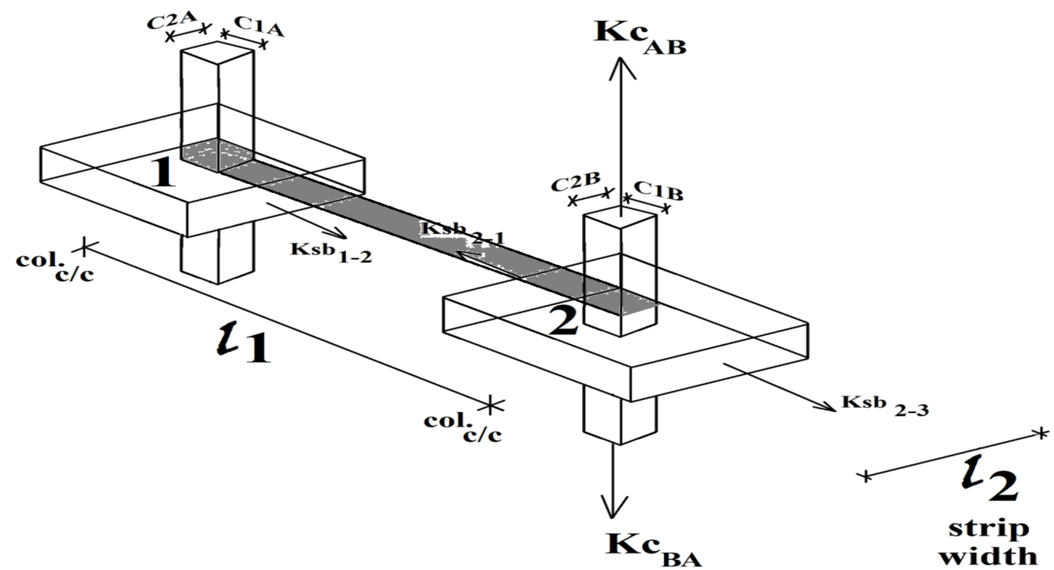
$$Ksb_{2-1} = K_{BA}\frac{4700\sqrt{f'c}\ \ (C_t\,\frac{b_w h^3}{12})}{L1}$$



Where h= total height of beam.

To get Ct, the following figure should be used:

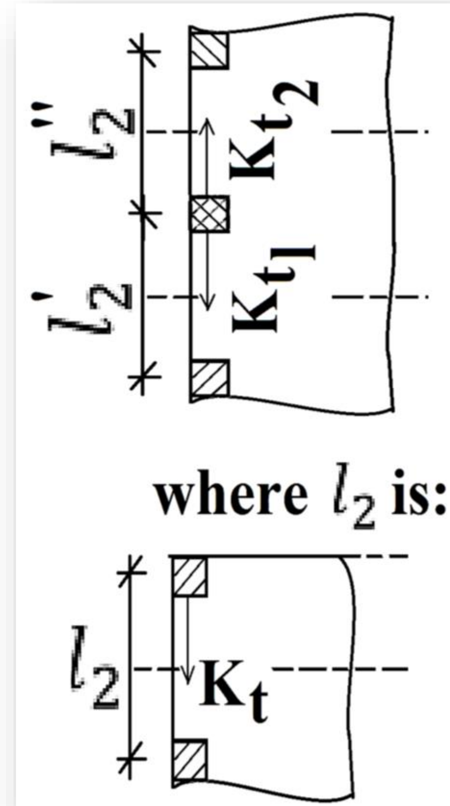




Torsional stiffness, *Kt*.

$$K_t = \sum \frac{9\,E_{cs}\,C}{l_2\left(1-\frac{c_2}{l_2}\right)^3}$$

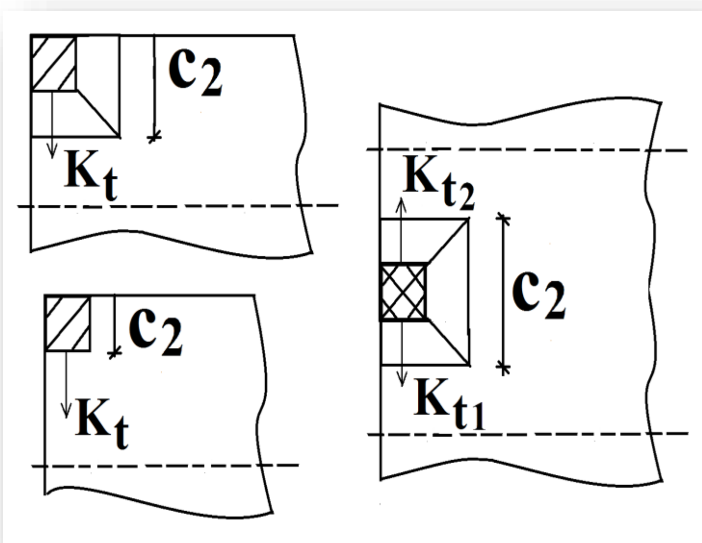
Note: $(1-\frac{c_2}{l_2})$ is magnification factor

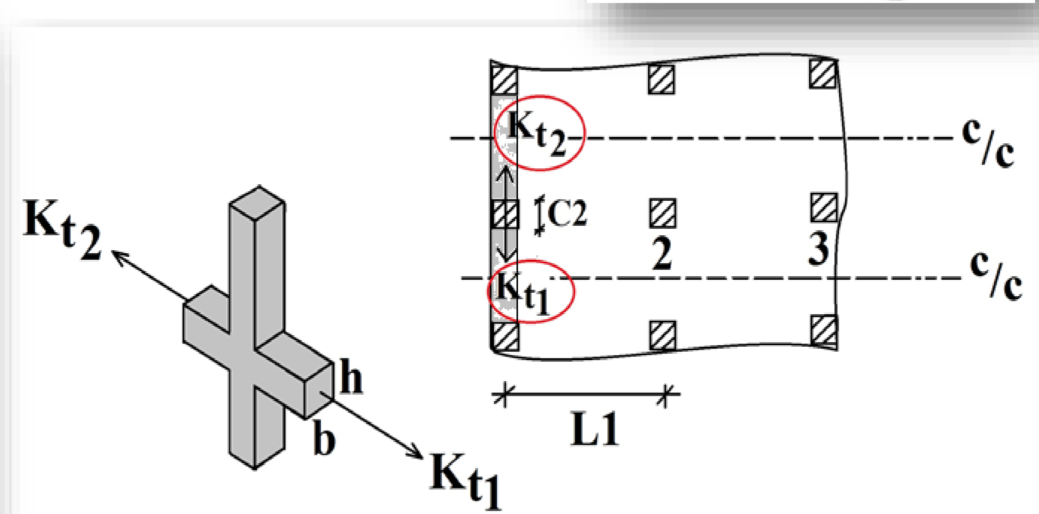


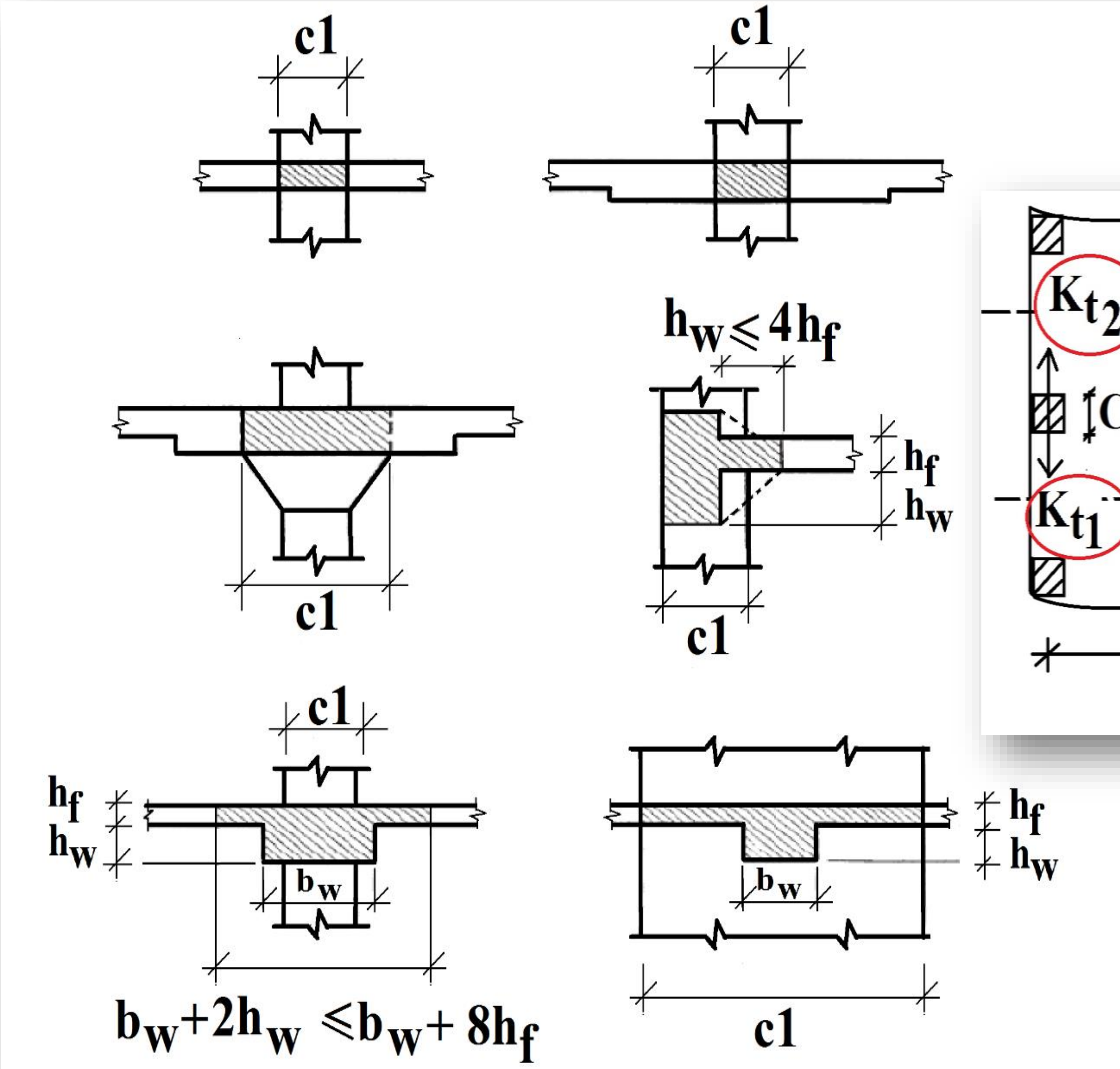
Where

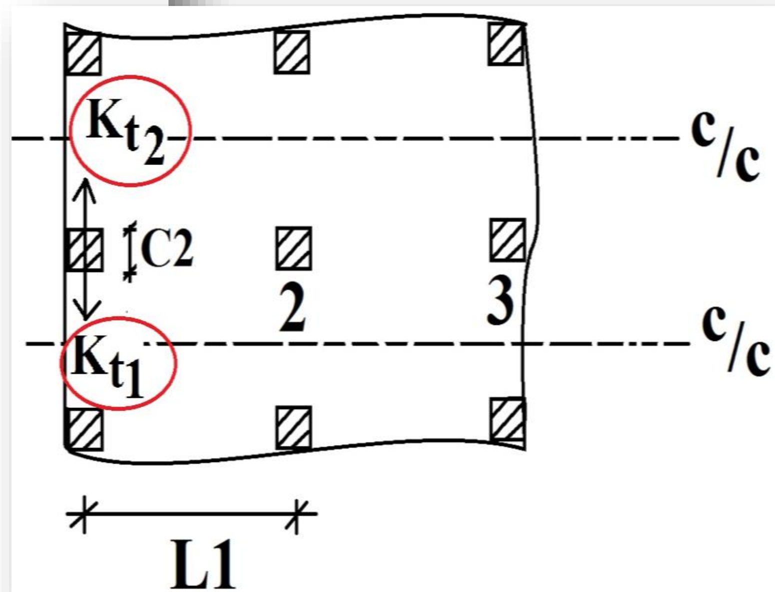
c2= column dimension which is perpendicular to the strip.

C=torsional constant (max value), $C = \sum(1-0.63\frac{x}{y})\frac{x^3y}{3}$, where x is the smaller dimension and y is the greater one.









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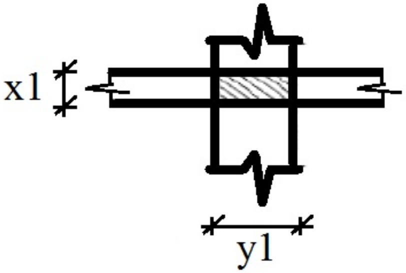
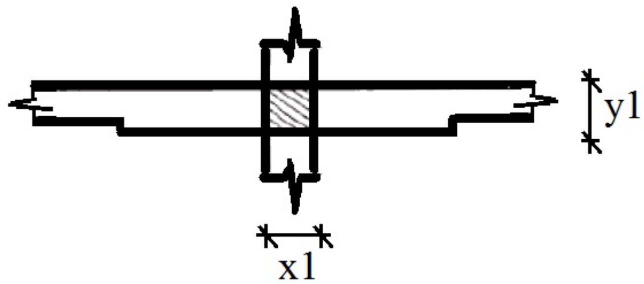
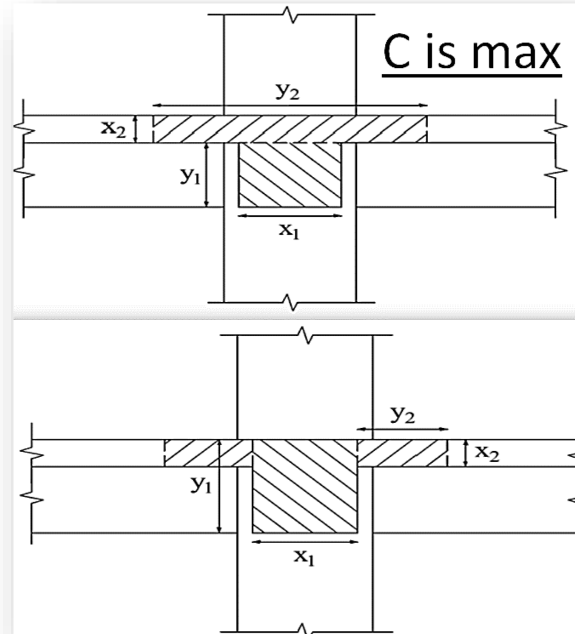
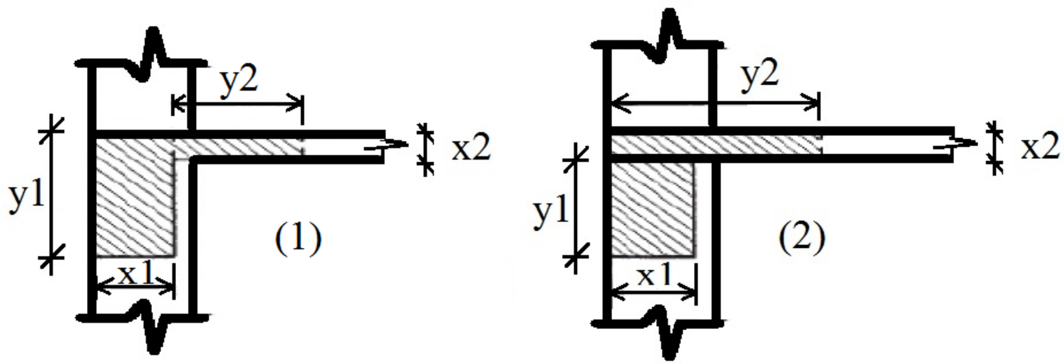
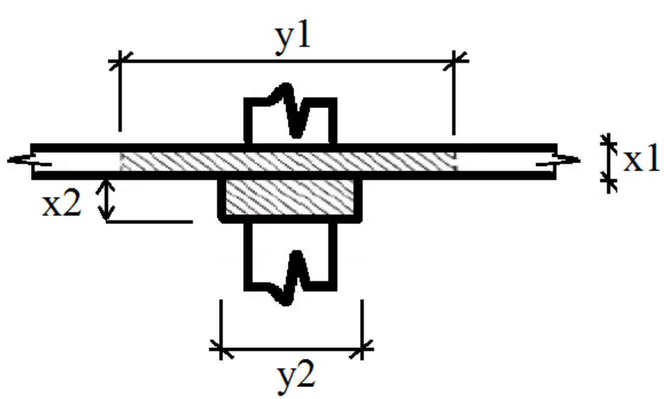








Use larger C computed in (1) and (2)



$$K_t = \sum \frac{9\, E_{cs}\, C}{l_2 \left(1 - \frac{c_2}{l_2}\right)^3}$$

Note: $(1 - \frac{c_2}{l_2})$ is magnification factor

Where

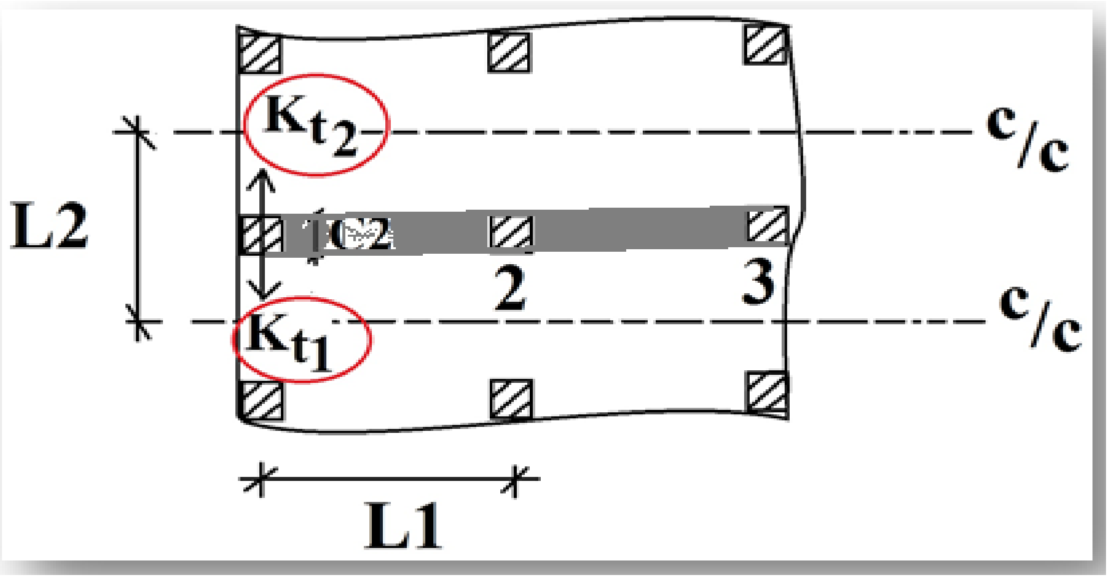
c2= column dimension which is <u>perpendicular</u> to the strip.

C=torsional constant (max value), $C = \sum(1 - 0.63\frac{x}{y})\frac{x^3 y}{3}$

, where x is the smaller dimension and y is the greater one.

<u>Note:</u> when there is a parallel beam in the strip under consideration, K_t will be modified through K_{tm} by:

$$K_{tm} = K_t \frac{I_{sb}}{I_s}$$

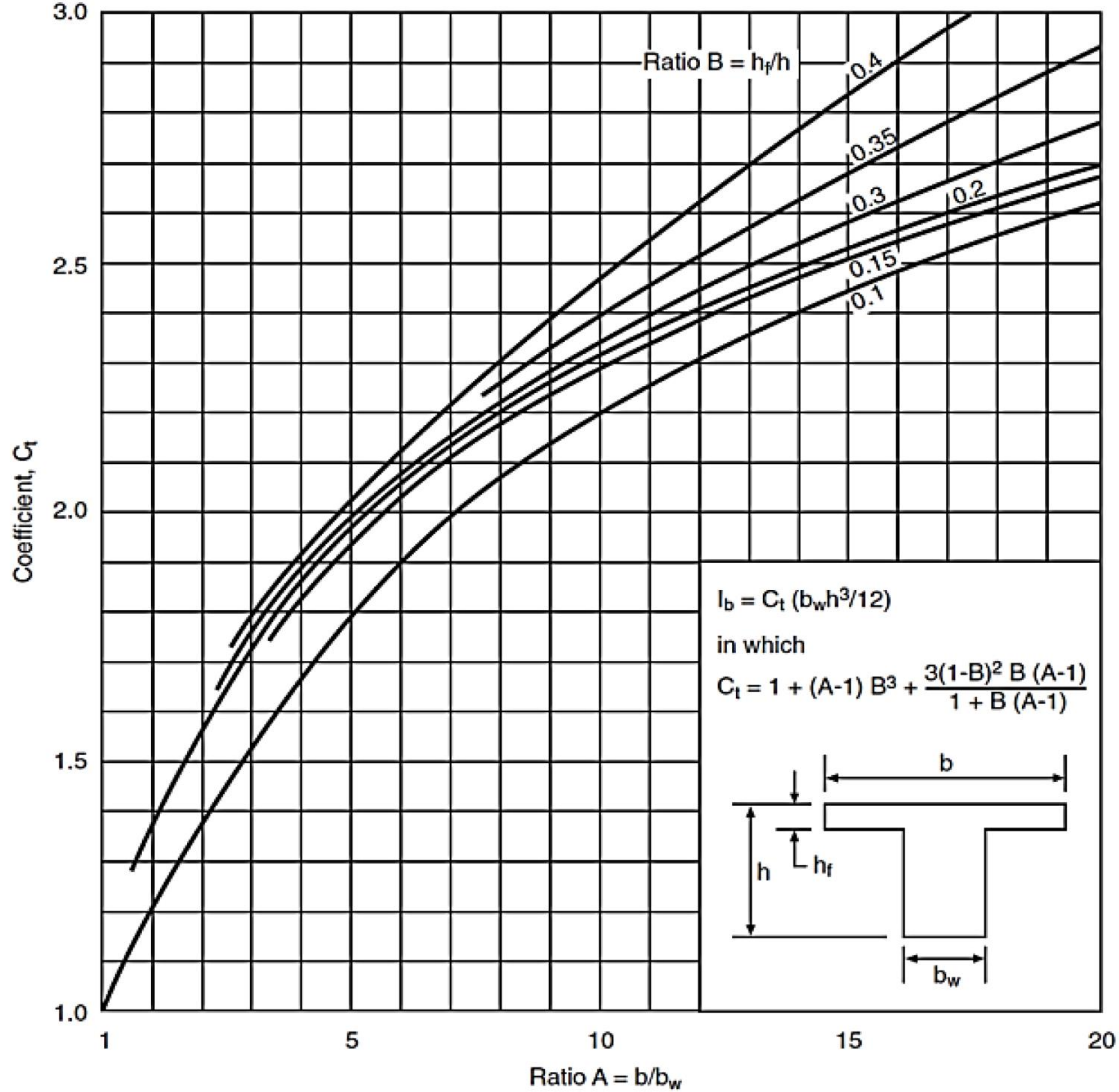


where $$I_{sb} = \frac{b_{sb}\, h^3}{12} C_t$$

and $$I_s = \frac{L_2\, h_f^3}{12}$$

where I_{sb} is the moment of inertia of the slab and beam together, and I_s is the moment of inertia of the slab neglecting the beam section.

C_t



Ratio B = h_f/h
0.4
0.35
0.3
0.2
0.15
0.1
3.0
2.5
2.0
1.5
1.0
Coefficient, C_t
$I_b = C_t\,(b_w h^3/12)$
in which
$C_t = 1 + (A-1)\,B^3 + \dfrac{3(1-B)^2\,B\,(A-1)}{1 + B\,(A-1)}$
b
h
h_f
b_w
1
5
10
15
20
Ratio A = b/b_w

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Flexural stiffness of actual column Kc

$$Kc_{AB} = K_{AB}\frac{4700\sqrt{f'c}\ \ I_c}{L_c} \qquad (top\ column)$$

$$Kc_{BA} = K_{BA}\frac{4700\sqrt{f'c}\ \ I_c}{L_c} \qquad (bottom\ column)$$

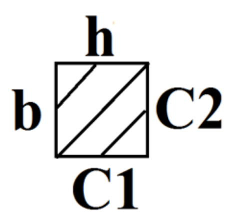
$$I_c = \frac{b\ h^3}{12}$$

$where$:

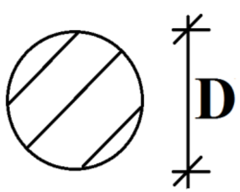
$b\&h =\ perpendicular\ \&\ parallel$ $sectional\ dimensions\ of\ column$

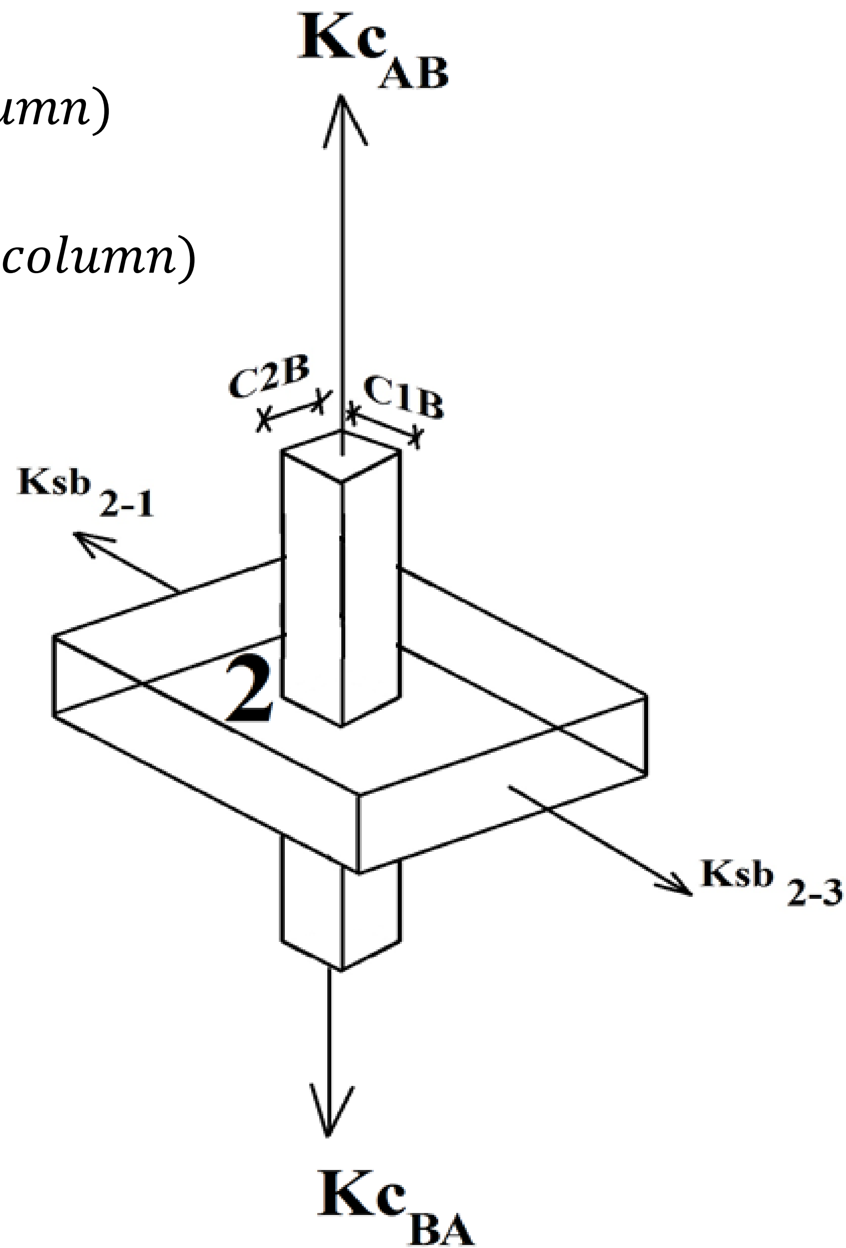
$L_c = column\ c/c\ \ length$

$$\mathbf{Ic = \frac{bh^3}{12} = \frac{C2.C1^3}{12}}$$



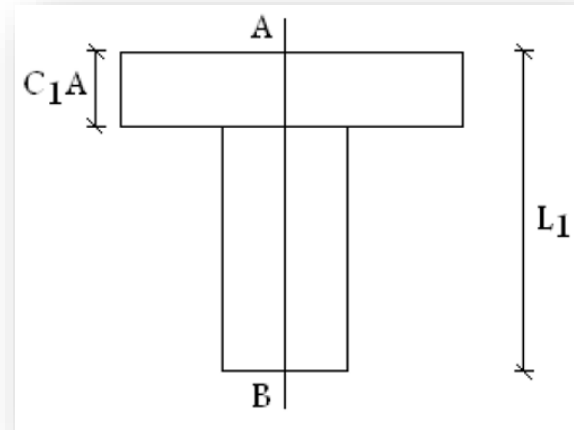
$$\mathbf{Ic = \frac{\pi\ .\ D^4}{64}}$$

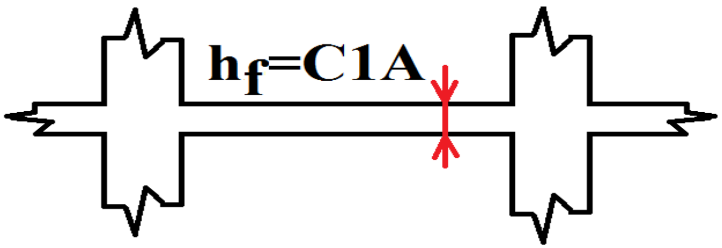


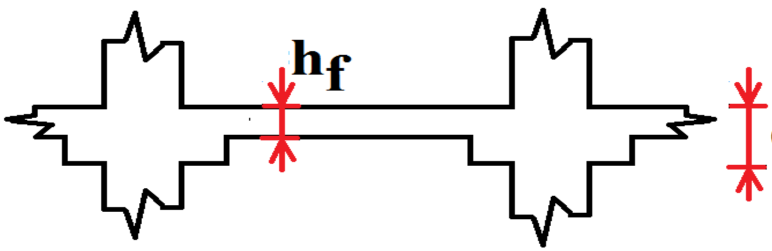


Notes on (C1A) considering for columns (Table 3)

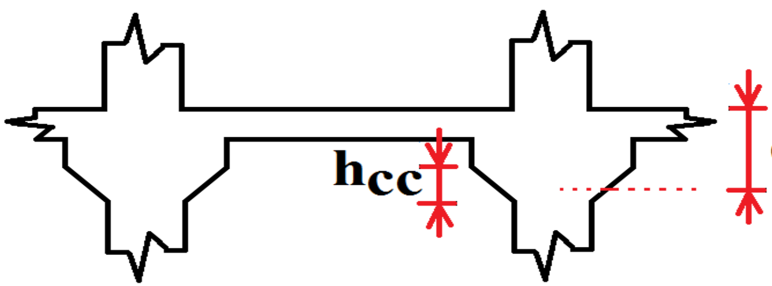
1- When the column is not connected to a beam



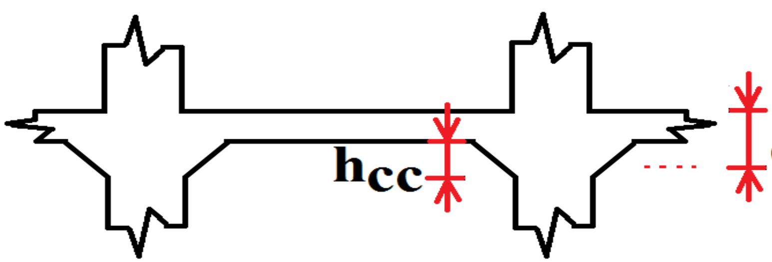




$C1A=h_f+h_f/4$

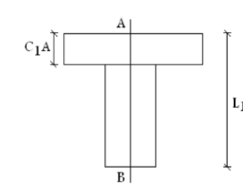


$C1A=h_f+h_f/4+h_{cc}/2$



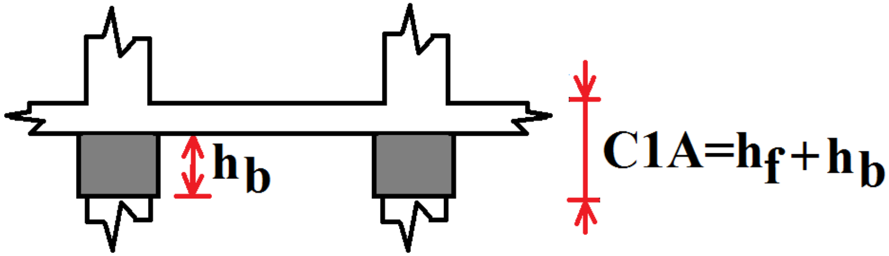
$C1A=h_f+h_{cc}/2$

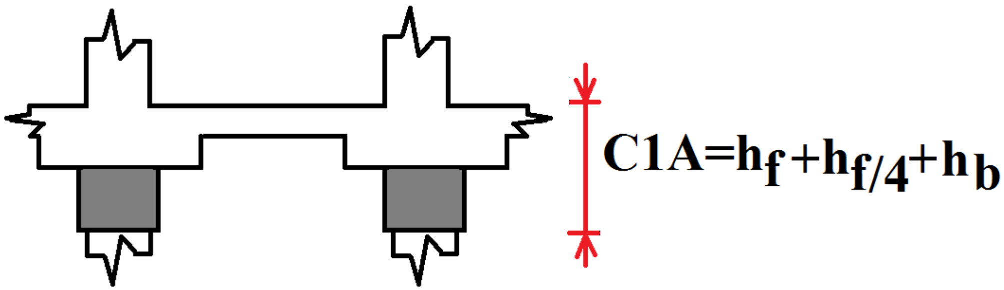
Table 3 Coefficients for columns with variable moment of inertia

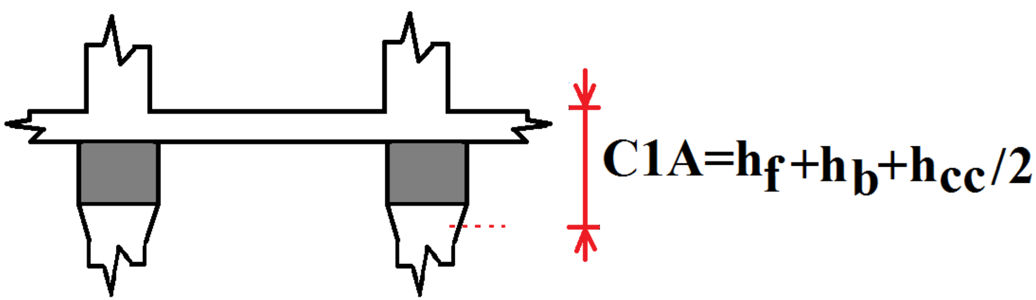


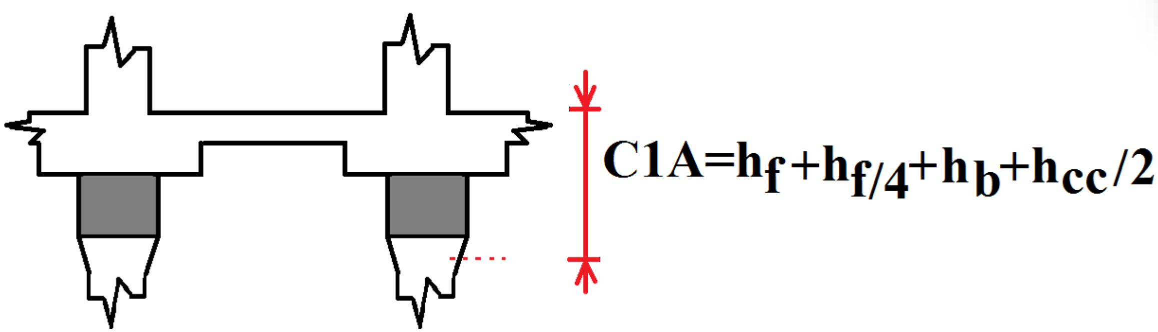
Slab Depth C_1A/L_1	Uniformly Load F.E.M= coef. ($w\,L_2.L_1^2$)		Stiffness Factors		Carry-over Factors	
	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.083	0.083	4.00	4.00	0.500	0.500
0.05	0.100	0.075	4.91	4.21	0.496	0.579
0.10	0.118	0.068	6.09	4.44	0.486	0.667
0.15	0.135	0.060	7.64	4.71	0.471	0.765
0.20	0.153	0.053	9.69	5.00	0.452	0.875
0.25	0.172	0.047	12.44	5.33	0.429	1.000

2- When the column is connected to a beam









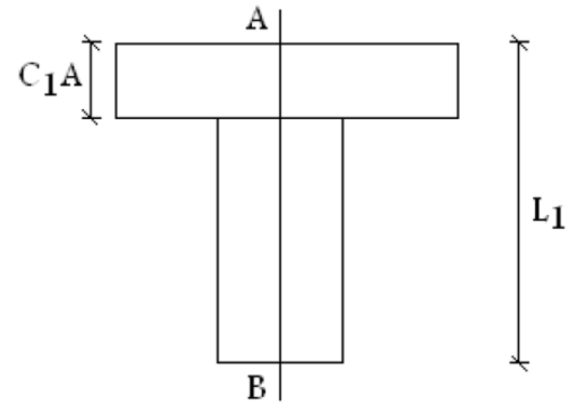
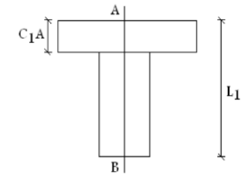


Table 3 Coefficients for columns with variable moment of inertia



Slab Depth C_1A/L_1	Uniformly Load F.E.M= coef. ($w\,L_2.L_1^2$)		Stiffness Factors		Carry-over Factors	
	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	*0.083*	*0.083*	*4.00*	*4.00*	*0.500*	*0.500*
0.05	*0.100*	*0.075*	*4.91*	*4.21*	*0.496*	*0.579*
0.10	*0.118*	*0.068*	*6.09*	*4.44*	*0.486*	*0.667*
0.15	*0.135*	*0.060*	*7.64*	*4.71*	*0.471*	*0.765*
0.20	*0.153*	*0.053*	*9.69*	*5.00*	*0.452*	*0.875*
0.25	*0.172*	*0.047*	*12.44*	*5.33*	*0.429*	*1.000*

Flexural stiffness of equivalent column (Kec):

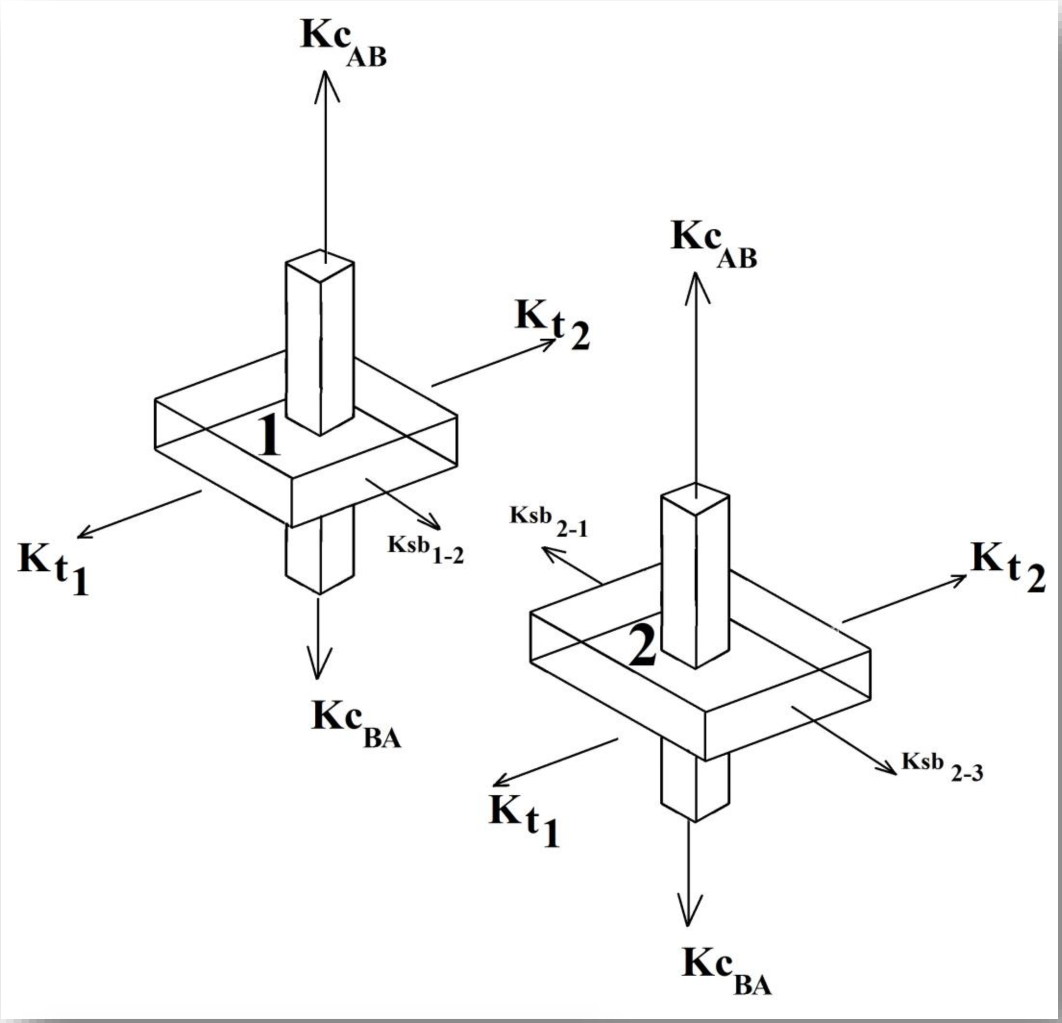
$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t}$$

Distribution Factor (DF):

$$DF_{1-2} = \frac{K_{sb1-2}}{K_{sb1-2} + K_{ec}}$$

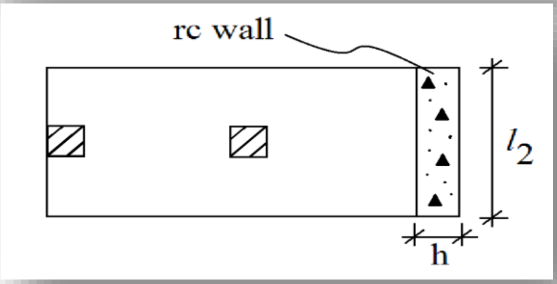
$$DF_{2-1} = \frac{K_{sb2-1}}{K_{sb2-1} + K_{sb2-3} + K_{ec}}$$

$$DF_{2-3} = \frac{K_{sb2-3}}{K_{sb2-1} + K_{sb2-3} + K_{ec}}$$



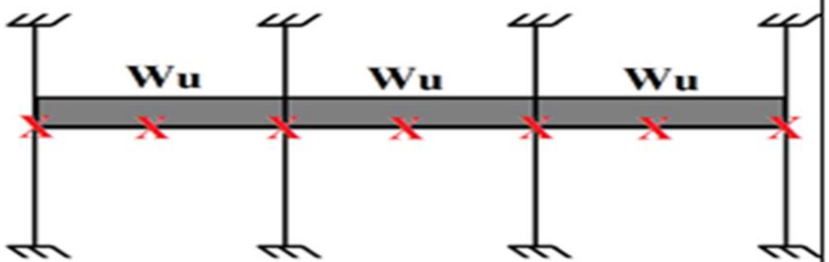
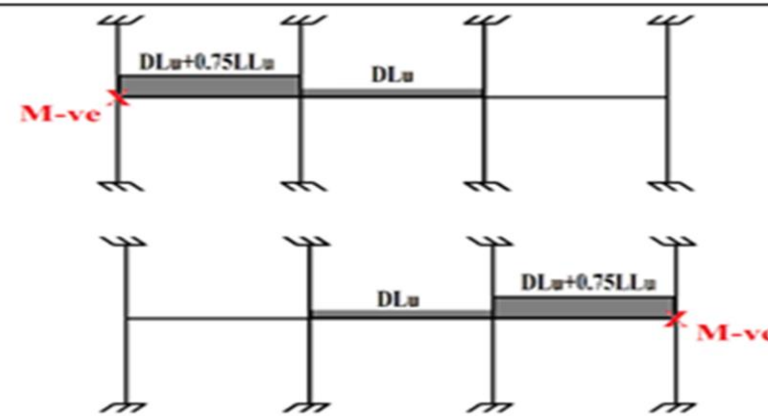
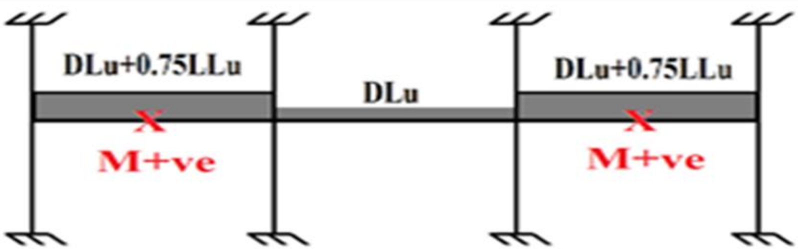
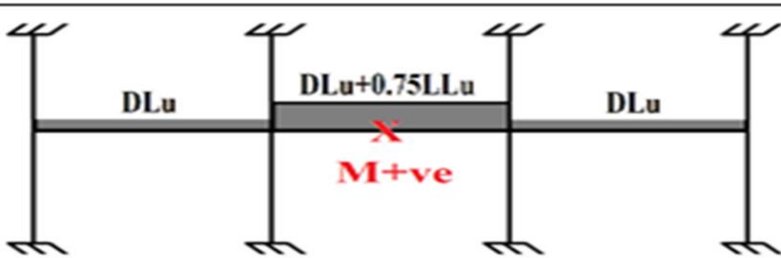
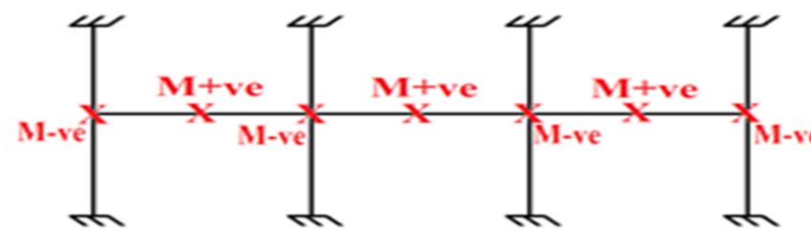
<u>Note:</u> when slab is supported by a rc wall, DF=0

$$Kc = K_{AB}\frac{4700\sqrt{f\prime c}\ I_c}{L_1} \qquad I_c = \frac{l_2\, h^3}{12}$$



Loading Arrangement

If LL is known and LL≤0.75DL then only full load case (case 1) is used, otherwise (LL>0.75DL) the five cases (from 1 to 5) should be summarized in one case (max case):

Pattern	Case	Pattern	Case
	1		4
	2		5
	3		Summery (max)